# LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2024**

**PMT3MC03 – MATHEMATICAL PROGRAMMING**

Date: 12-11-2024 Dept. No. Max. : 100 Marks

Time: 01:00 pm-04:00 pm

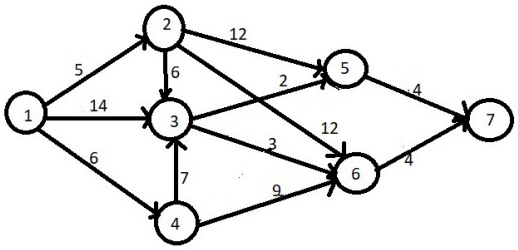
| | | |
|---|---|---|
| | **SECTION A – K1 (CO1)** | |
| | **Answer ALL the questions** | **(5 x 1 = 5)** |
| **1** | **Answer the following** | |
| a) | Define objective function. | |
| b) | Explain about under achievement and over achievement. | |
| c) | Name any two examples of dynamic programming problem. | |
| d) | Explain about Non-linear programming. | |
| e) | Give the advantages and disadvantages of greedy heuristic. | |
| | **SECTION A – K2 (CO1)** | |
| | **Answer ALL the questions** | **(5 x 1 = 5)** |
| 2 | **MCQ** | |
| a) | The study of effect of changes in parameters on the optimal solution is called<br>i. sensitivity analysis ii. post-optimality analysis<br>iii. both i and ii iv. none of these | |
| b) | Which of the following method prioritize the goals in the order of importance?<br>i. Pre-emptive method ii. Two phase method<br>iii. Simplex method iv. none | |
| c) | Which of the following is not a integer programming problem?<br>i. Branch and bound method ii. Cutting plane method<br>iii. Both i and ii iv. None of these | |
| d) | When was the Kuhn Tucker conditions first published?<br>i. 1951 ii. 1952<br>iii. 1953 iv. 1954 | |
| e) | The Greedy Heuristic are usually<br>i. Single variable problem ii. Double variable problem<br>iii. Multivariable problem iv. All the above | |
| | **SECTION B – K3 (CO2)** | |
| | **Answer any THREE of the following** | **(3 x 10 = 30)** |
| 3 | A patient consults a doctor to check his ill health. Doctor examines and advised him that he is having a deficiency of vitamin A and D. The doctor advises him to take vitamin A and D for a period to recover his health. He prescribes tonic X and Y which are having vitamin A and D in certain composition. Doctor tells the patient to consume atleast 40 units of A and 50 units of D daily. Costs of tonic and proportions of vitamins are given in the table. Apply graphical method to minimize the costs of tonics | |

| Vitamins | Tonic | | Daily consumption |
|---|---|---|---|
| | X | Y | |
| A | 2 | 4 | 40 |

| | | | |
|---|---|---|---|
| D | 3 | 2 | 50 |
| Cost in Rs | 5 | 3 | |

4 A manufacturing company produces A and B each product must be processed through two department. Department 1 has 30 hours of production capacity per week and department 2 has 60 hours per week, each unit of product A requires 2 hours in department 1 and 6 hours in department 2. Each unit of product B requires 3 hours in dept 1 and 4 hours in dept 2. The management has set the following priorities $P_1$: Minimize the underachievement of joint production of 10 units, $P_2$: Minimize the underachievement of 7 units of product B, $P_3$: Minimize the overachievement of 8 units of product A. Demonstrate the problem as goal programming problem.

5 The network below provides the possible routes between the starting city at node 1 and the destination city at node 7. The routes pass through intermediate cities designated by nodes 2 to 6.




Calculate the shortest route from city 1 to city 7.

6 Determine the value of $x, y, z$ that maximize or minimize the function $f(x,y,z)=xy+10x-x^2-y^2-z^2$.

7 Use random walk heuristic to solve the problem $min\, F(x), x\epsilon S=\{1,2,\ldots 8\}$

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $F(x)$ | 90 | 60 | 50 | 80 | 100 | 40 | 20 | 70 |

the random numbers are 0.4123, 0.2014, 0.0829, 0.5819, 0.3717.

## SECTION C – K4 (CO3)

**Answer any TWO of the following** **(2 x 12.5 = 25)**

8 *Determine the optimum solution using simplex method* ¿ $max\, Z=5x+4y$, subject to the constraints $6x+4y\le 24, x+2y\le 6, -x+y\le 1, y\le 2, x, y\ge 0$.

9 a. Harrison Electric Produces two products popular with home renovators, old fashioned chandeliers and ceiling fans. Both chandeliers and fans require a two-step production process involving wiring and assembly. It takes about 2hrs to wire a chandelier and 3 hrs to wire a fan. Final assembly of the chandelier and fan require 6 and 5 hrs respectively. The production capability is such that only 12 hrs of wiring and 30 hrs of assembly time are available. Each chandelier produced nets the firm \$7 and each fan \$6. The Harrison's management wants to achieve the following goals with the given priorities: $P_1$: Reach a profit as much above \$30 as possible, $P_2$: Fully use wiring department hours available, $P_3$: Avoid assembly department overtime, $P_4$: Produce atleast 7 ceiling fans. Formulate and solve the above goal programming problem using graphical method.

b.Using dynamic programming estimate the shortest distance from city 1 to city 8

| | 2 | 3 | 4 |
|---|---|---|---|
| 1 | 20 | 35 | 17 |

| | 5 | 6 | 7 |
|---|---|---|---|
| 2 | 18 | 7 | 11 |
| 3 | 99 | 10 | 45 |
| 4 | 28 | 24 | 13 |

| | 8 |
|---|---|
| 5 | 9 |
| 6 | 4 |
| 7 | 12 |

(7+5.5)

10 Use lagrangian multipliers to estimate the optimal solution to $f=2x^2+y^2+3z^2+10x+8y+6z-100$, subject to $x+y+z=20$.

11 Solve the optimization problem using tabu search algorithm for the random numbers 0.915, 0.3118, 0.4129, 0.1816, 0.5001, 0.6814 $min\, F(x), x\epsilon S, S=\{1,2,\ldots 8\}$

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $F(x)$ | 90 | 60 | 50 | 80 | 100 | 40 | 30 | 70 |

## SECTION D – K5 (CO4)

**Answer any ONE of the following** **(1 x 15 = 15)**

12 a. TopAd, a new advertising agency with 10 employees, has received a contract to promote a new product. The agency can advertise by radio and television. The following table gives the number of people reached by each type of advertisement and the cost and labour requirements.

| | Data/min advertisement | |
|---|---|---|
| | Radio | Television |
| Exposure (in millions of persons) | 4 | 8 |
| Cost (in thousands of dollars) | 8 | 24 |
| Assigned employees | 1 | 2 |

The contract prohibits TopAd from using more than 6 minutes of radio advertisement. Additionally, radio and television advertisements need to reach at least 45 million people. TopAd has a budget goal of $100,000 for the project. TopAd's management assumes that the exposure goal is important than the budget goal. How many minutes of radio and television advertisement should TopAd use – Justify your answer using graphical method.

b. Determine the optimal solution for the discrete single variable problem using greedy search $min\, F(x), x = S = \{1,2,\ldots 9\}$

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $F(x)$ | 100 | 80 | 50 | 40 | 70 | 70 | 20 |

(8+7)

13 Use branch and bound method to evaluate $max\, Z = 5x + 4y$ subject to the constraints $x + y \le 5, 10x + 6y \le 45$, where $x, y$ are nonnegative integers.

## SECTION E – K6 (CO5)

**Answer any ONE of the following** **(1 x 20 = 20)**

14 Evaluate the optimal solution of the following linear programming problem $max\, Z = 40x + 30y$ subject to the constraints; $x + y \le 12$, $2x + y \le 16$.

i. Applying the sensitivity analysis find the solution of the given linear programming problem, if the right-hand side vector $\begin{bmatrix} 12 \\ 16 \end{bmatrix}$ changes to $\begin{bmatrix} 15 \\ 17 \end{bmatrix}$.

ii. Also find out how far the new resource can be decreased.

15 Using Karush Kuhn Tucker condition formulate the optimal solution so as to $max\, f = -x^2 - y^2 - z^2 + 4x + 6y$ subject to the constraints $x + y \le 2$, $2x + 3y \le 12$ where $x, y \ge 0$.

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